Math 53, Discussions 116 and 118 **2022-8-26**

Parametric curves and their tangents

Conceptual questions

Question 1. In lecture, you saw that $x = \cos t, y = \sin t$ parametrizes a circle. Where is the center of the circle, and what is the circle's radius? What point on the circle corresponds to $t = 0$? As t increases, is the circle traced clockwise or counterclockwise? How long does it take to trace out the circle exactly once?

Then answer the same questions for these other circles:

- $x = 3\sin t, y = 3\cos t$
- $x = \cos(2t), y = \sin(2t)$
- $x = 4 + \cos t, y = -3 + \sin t$

Question 2. Beware: the following parametric curves are *not* circles. (What are they?)

- $x = 2\cos t, y = 5\sin t$
- $x = \sin t, y = \cos(2t)$. (Hint: use the double angle formula to find a Cartesian equation.)

Question 3. Suppose that a parametric curve $x = f(t), y = g(t)$ satisfies $g'(3) = 0$. What can you conclude (if anything) about the tangent to the curve at $t = 3$?

Computations

Problem 1. This is a further exploration of some ideas from Question 1, so that you can better understand how to transform shapes algebraically. Let C be the curve described by the equation $x^2 + y^2 = 5$. If we take C and

(1) shift it by 2 units in the positive y direction (i.e. upwards),
(2) and then stretch it by a factor of 3 in the x direction (i.e. horizontally),

what Cartesian equation describes the resulting shape?

Next, come up with a parametrization $x = f(t)$, $y = g(t)$ for the starting shape C, and then a parametrization for the shape obtained after applying the transformations.

Problem 2. Find a Cartesian equation for the parametric curve $x = t^3 + t$, $y = t^2 + 2$. Hint: compute x^2.

Problem 3. There are two points on the curve

$$x = 2t^2, y = t - t^2, -\infty < t < \infty$$

where the tangent line passes through the point $(10, -2)$. Find these two points.